**PROGRESSION THROUGH CALCULATIONS FOR ADDITION**

**Early addition**

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

 

**Number lines**

They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.

3 + 2 = 5

+1

+1

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0 1 2 3 4 5 6 7 8 9

Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13

+1

+1

+1

+1

+1

 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Bead strings can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

Children will then begin to use ‘empty number lines’ themselves starting with the larger number and counting on.

First counting on in tens and ones.

34 + 23 = 57

+10

+10

+1

+1

+1

34 44 54 55 56 57

Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

34 + 23 = 57

+10

+10

+3

 34 44 54 57

Followed by adding the tens in one jump and the units in one jump.

34 + 23 = 57

+20

+3

 34 54 57

Bridging through ten can help children become more efficient.

37 + 15 = 52

+10

+2

+3

37 47 50 52

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.

* Count on from the largest number irrespective of the order of the calculation.

38 + 86 = 124

+30

+4

+4

86 116 120 124

* Compensation

49 + 73 = 122

+50

-1

73 122 123

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

**Partitioning**

**Expanded column method**

Adding the least significant digits first

1. 267

 + 24 + 85

 1 1 (20+ 4) 12 ( 80 + 5)

 80 (60 + 7) 67 (60 + 7)

 91 200

 352

**Column method**

From this, children will begin to carry below the line.

 **1 1 11**

625 783 367

 + 48 + 42 + 85

 673 825 452

Children should extend the carrying method to numbers with at least four digits.

 1 1 1 1 1

 587 3587

 + 475 + 675

 1062 4262

Children should extend the carrying method to number with any number of digits.

 **111 111 121**

7648 6584 42

 + 1486 + 5848 6432

 9134 12432 786

 3

 + 4681 11944

**PROGRESSION THROUGH CALCULATIONS FOR Subtraction**

**Early subtraction**

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.



**Number Lines**

They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line.

6 – 3 = 3

-1

-1

-1

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0 1 2 3 4 5 6 7 8 9 10

The number line should also be used to show that 6 - 3 means the ‘difference between

6 and 3’ or ‘the difference between 3 and 6’ and how many jumps they are apart.

0 1 2 3 4 5 6 7 8 9 10

Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

13 – 5 = 8

-1

-1

-1

-1

-1

 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Bead strings can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

13 – 5 = 8

Children will begin to use empty number lines to support calculations.

**Counting back**

* First counting back in tens and ones.

47 – 23 = 24

-1

-1

-1

- 10

- 10

 24 25 26 27 37 47

* Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4).

47 – 23 = 24

-10

-10

-3

 24 27 37 47

* Subtracting the tens in one jump and the units in one jump.

47 – 23 = 24

-20

-3

 24 27 47

* Bridging through ten can help children become more efficient.

42 – 25 = 17

-3

-20

-2

 17 20 22 42

**Counting on and finding the difference**

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

82 - 47

+10

+10

+10

+1

+1

+1

+1

+1

 47 48 49 50 60 70 80 81 82

Children will continue to use empty number lines with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

**Partitioning and decomposition**

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

 89 = 80 + 9

 - 57 50 + 7

 30 + 2 = 32

*Initially, the children will be taught using examples that do not need the children to exchange.*

**From this the children will begin to exchange.**

 71 = =

 - 46

Step 1 70 + 1

The calculation should be read as e.g. take 6 from 1.

 - 40 + 6

 Step 2 60 + 11

 - 40 + 6

 20 + 5 = 25

This would be recorded by the children as

 60

 70 + 11

 - 40 + 6

 20 + 5 = 25

*Children should know that units line up under units, tens under tens, and so on.*

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

102 – 89 = 13

+10

+1

+2

 89 90 100 102

 754 =

-86

Step 1 700 + 50 + 4

 - 80 + 6

Step 2 700 + 40 + 14 *(adjust from T to U)*

 - 80 + 6

Step 3600 + 140 + 14 *(adjust from H to T)*

 - 80 + 6

 600 + 60 + 8 = 668

This would be recorded by the children as

 600 140

 700 + 50 + 14

 - 80 + 6

 600 + 60 + 8 = 668

**Decomposition**

 614 1

**//**

 754

 - 86

 668

*For example:*

 £8.95 = 8 + 0.9 + 0.05 leading to

-£4.38 - 4 + 0.3 + 0.08

  1

= 8 + 0.8 + 0.15 *(adjust from T to U)* 8.85

- 4+0.3+ 0.08 - 4.38

 4 + 0.5 + 0.07

 = £4.57

Alternatively, children can set the amounts to whole numbers, i.e. 895 – 438 and convert to pounds after the calculation.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

511 – 197 = 314

+300

+3

+11

 197 200 500 511

Step 1 754 = 700 + 50 + 4

-286 - 200 + 80 + 6

Step 2 700 + 40 + 14 *(adjust from T to U)*  -200 + 80 + 6

Step 3600 + 140 + 14 *(adjust from H to T)*

 - 200 + 80 + 6

 400 + 60 + 8 = 468

This would be recorded by the children as

 600 140

 700 + 50 + 14

-200 + 80 + 6

 400 + 60 + 8 = 468

**Decomposition**

 614 1

**//**

 754

 - 286

 468

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

1209 – 388 = 821

+800

+12

+9

 388 400 1200 1209

 5 13 1

 6467

 - 2684

 3783

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

+1000

3002 – 1997 = 1005

+3

+2

 1997 2000 3000 3002

**PROGRESSION THROUGH CALCULATIONS FOR Multiplication**

**Early Multiplication**

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.

 

**Repeated addition and number lines**

Children will develop their understanding of multiplication and use jottings to support calculation:

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5

5

5

5

 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

and on a bead sting:

5 x 3 = 5 + 5 + 5

5

5

5

Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.

5

5

5

 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

3

3

3

3

3

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines or bead stings to support their understanding.

6

6

6

6

0 6 12 18 24

6

6

6

6

* **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

5 x 3 = 15

3 x 5 = 15

Children will continue to use:

* **Repeated addition**

4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines or bead stings to support their understanding.

6

6

6

6

0 6 12 18 24

6

6

6

6

**Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

9 x 4 = 36

9 x 4 = 36

**Partitioning**

When teaching partitioning ensure that the children know and understand their ten times tables and what happens when you multiply a number by ten.

38 x 5 = (30 x 5) + (8 x 5)

 = 150 + 40

 = 190

Children will continue to use arrays where appropriate leading into the grid method of multiplication.

 x 10 4

(6 x 10) + (6 x 4)

24

60

 60 + 24

 84

6

**Grid method**

**TU x U**

(Short multiplication – multiplication by a single digit)

23 x 8

Children will approximate first

23 x 8 is approximately 25 x 8 = 200

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x |  20 |  3 |  |  |  |
| 8 | 160 |  24 |  |  |  160 |
|  |  |  |  |  |  + 24 |
|  |  |  |  |  |  184 |

**HTU x U**

346 x 9

Children will approximate first

346 x 9 is approximately 350 x 10 = 3500

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x |  300 |  40 |  6 |  |  |  |
| 9 | 2700  | 360 | 54 |  |  |  2700 |
|  |  |  |  |  |  | + 360 |
|  |  |  |  |  |  | + 54 |
|  |  |  |  |  |  |  31 1 4 |

 1 1

**ThHTU x U**

4346 x 8

Children will approximate first

4346 x 8 is approximately 4346 x 10 = 43460

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  X |  4000 |  300 |  40 |  6 |  |  |  |
| 8 | 32000 |  2400 |  320 |  48 |  |  |  32000 |
|  |  |  |  |  |  |  | + 2400 |
|  |  |  |  |  |  |  | + 320 |
|  |  |  |  |  |  |  | + 48 |
|  |  |  |  |  |  |  |  34768 |

4.92 x 3

Children will approximate first

4.92 x 3 is approximately 5 x 3 = 15

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x |  4 | 0.9 | 0.02 |  |  |  |
| 3 | 12 | 2.7 | 0.06 |  |  |  12 |
|  |  |  |  |  |  | + 0.7 |
|  |  |  |  |  |  | + 0.06 |
|  |  |  |  |  |  |  12.76 |

**TU x TU**

(Long multiplication – multiplication by more than a single digit)

72 x 38

Children will approximate first

72 x 38 is approximately 70 x 40 = 2800

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x |  70 |  2 |  |  |  |
| 30 | 2100 | 60 |  |  |  2100 |
| 8 |  560 | 16 |  |  | + 560 |
|  |  |  |  |  | + 60 |
|  |  |  |  |  | + 1 6 |
|  |  |  |  |  |  2736 |

  1

e.g. 4.9 x 3

Children will approximate first

4.9 x 3 is approximately 5 x 3 = 15

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x |  4 | 0.9 |  |  |  |
| 3 |  12 | 2.7 |  |  |  12 |
|  |  |  |  |  | + 2.7 |
|  |  |  |  |  |  14.7 |

**HTU x TU**

372 x 24

Children will approximate first

372 x 24 is approximately 400 x 25 = 10000

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x |  300 |  70 |  2 |  |  |  |
| 20 | 6000 |  1400 |  40 |  |  |  6000 |
| 4 | 1200 |  280 |  8 |  |  | + 1400 |
|  |  |  |  |  |  | + 1200 |
|  |  |  |  |  |  | + 280 |
|  |  |  |  |  |  | + 40 |
|  |  |  |  |  |  | + 8 |
|  |  |  |  |  |  |  8928 |

 1

**Long Multiplication – Traditional Method**

372 x 24

As before, children should approximate first:

372 x 24 is approximately 400 x 25 = 10000

 372

x 24

1488

 1 2

 +7440

 1

\_\_\_\_\_\_\_

 =8928

 1

**PROGRESSION THROUGH CALCULATIONS FOR Division**

**Early division**

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.



**Y2**

Children will develop their understanding of division and use jottings to support calculation

* **Sharing equally**

6 sweets shared between 2 people, how many do they each get?

* **Grouping or repeated subtraction**

There are 6 sweets, how many people can have 2 sweets each?

**Number line**

12 ÷ 3 = 4

 0 1 2 3 4 5 6 7 8 9 10 11 12

 3 3 3 3

The bead sting will help children with interpreting division calculations such as 10 ÷ 5 as ‘how many 5s make 10?’

**Repeated subtraction**

24 ÷ 4 = 6

 0 4 8 12 16 20 24

Children should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1

 4 4 4

0 1 5 9 13

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

72 ÷ 5

 -2 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5

 0 2 7 12 17 22 27 32 37 42 47 52 57 62 67 72

Moving onto:

 -50

-5

-5

-5

-5

r2

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1

1

1

1

10

0 2 7 12 17 22 72

Chunking

**Short division TU ÷ U**

72 ÷ 3

3 ) 72

 - 30 10x

 42

 - 30 10x

12

 - 6 2x

 6

 - 6 2x

 0

Answer : 24

Leading to subtraction of other multiples.

96 ÷ 6

 1 6

 6 ) 96

 - 60 10x

 36

 - 36 6x

0

 Answer : 16

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box)

**Short division HTU** ÷ **U**

196 ÷ 6

32 r 4

6 ) 196

 - 180 30x

 16

 - 12 2x

4

 Answer : 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

**Long division - Traditional Method**

**HTU** ÷ **TU**

972 ÷ 36

27

 36 ) 972

 -72

 252

Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as 3 2/10 which could then be written as 3 1/5 in its lowest terms.

Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other.

**By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.**

* **Children should be encouraged to approximate their answers before calculating.**
* **Children should be encouraged to check their answers after calculation using an appropriate strategy.**
* **Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.**
* **Children should be encouraged where possible to move on from more labour-intensive methods to enable them to get to grips with more complex compilations.**